Math Topics:
1. Elementary Operations with Numerical and Algebraic Fractions
2. Operations with Exponents and Radicals
3. Linear Equations and Inequalities
4. Polynomials and Polynomial Equations
5. Functions
6. Trigonometry
7. Logarithmic and Exponential Functions
A. Simplifying fractions (by reducing):

example: \( \frac{27}{36} = \frac{9 \cdot 3}{9 \cdot 4} = \frac{9}{4} \cdot \frac{3}{1} = \frac{3}{4} \) (note that you must be able to find a common factor—in this case 9—in both the top and bottom in order to reduce a fraction.)

example: \( \frac{3a}{12ab} = \frac{3a \cdot 1}{3a \cdot 4b} = \frac{3a}{4b} \cdot \frac{1}{1} = \frac{1}{4b} \)

(common factor: 3a)

1 to 8: Reduce:

1. \( \frac{13}{52} = \frac{1}{4} \)

2. \( \frac{26}{65} = \frac{2}{5} \)

3. \( \frac{3 + 6}{3 + 9} = \frac{9}{12} = \frac{3}{4} \)

4. \( \frac{6ax}{15by} = \frac{2ax}{5by} \)

5. \( \frac{5x + 7y}{7y} \)

6. \( \frac{5a + b}{5a + c} \)

7. \( \frac{x - 3}{3 - x} = -1 \)

8. \( \frac{4(x + 2)(x - 3)}{(x - 3)(x - 2)} = 4 \)

B. Equivalent fractions (equivalent ratios):

example: \( \frac{3}{4} \) is equivalent to how many eighths? \( \frac{3}{4} = \frac{3}{8} \)

example: \( \frac{6}{5a} = \frac{6b}{5ab} \)

example: \( \frac{2x + 2}{x + 1} = \frac{2(x + 1)}{x + 1} = 2 \)

example: \( \frac{x - 1}{x + 1} = \frac{(x + 1)(x - 1)}{(x + 1)(x + 1)} = \frac{x^2 - 3x + 2}{(x + 1)(x + 2)} \)

9 to 13: Complete:

9. \( \frac{4}{9} = \frac{72}{162} \)

10. \( \frac{3x}{7} = \frac{7y}{x} \)

11. \( \frac{x + 3}{x + 2} = \frac{(x - 1)(x + 2)}{(x + 1)(x + 2)} \)

12. \( \frac{30 - 15a}{15 - 15b} = \frac{(1 + b)(1 - b)}{(1 + b)(1 - b)} \)

13. \( \frac{x - 6}{6 - x} = \frac{-2}{1} \)

C. Finding the lowest common denominator (LCD) by finding the least common multiple (LCM) of all denominators:

example: \( \frac{5}{6} \) and \( \frac{8}{15} \).

First find LCM of 6 and 15:

6 = 2 \cdot 3
15 = 3 \cdot 5

LCM = 2 \cdot 3 \cdot 5 = 30, so

\( \frac{5}{6} \) and \( \frac{8}{15} \) are both equivalent to \( \frac{5}{6} \) and \( \frac{8}{15} \), so

\( \frac{5}{6} = \frac{25}{30} \) and \( \frac{8}{15} = \frac{16}{30} \)

example: \( \frac{3}{4} \) and \( \frac{1}{6a} \):

\( \frac{a}{3(x + 2)} \) and \( \frac{x}{6(x + 1)} \)

\( 3(x + 2) = 3 \cdot (x + 2) \)

\( 6(x + 1) = 2 \cdot 3 \cdot (x + 1) \)

so:

\( \frac{2}{3(x + 2)} = \frac{2}{2 \cdot 3(x + 1)(x + 2)} = \frac{4(x + 1)}{6(x + 1)(x + 2)} \)

and

\( \frac{ax}{6(x + 1)(x + 2)} \)

14 to 18: Find equivalent fractions with the lowest common denominator:

14. \( \frac{2}{3} \) and \( \frac{2}{9} \)

15. \( \frac{3}{x} \) and \( \frac{-1}{x + 1} \)

16. \( \frac{x}{3} \) and \( \frac{-1}{x + 1} \)

17. \( \frac{3}{x - 2} \) and \( \frac{4}{x - 2} \)

18. \( \frac{x}{15(x^2 - 2)} \) and \( \frac{7x + 1}{10x - 1} \)
D. Adding and subtracting fractions:
If denominators are the same, combine the numerators:

\[ \frac{3x}{y} - \frac{x}{y} = \frac{3x - x}{y} = \frac{2x}{y} \]

If denominators are different, find equivalent fractions with common denominators:

\[ \frac{a}{b} - \frac{a}{c} = \frac{2a}{4} - \frac{a}{4} = \frac{2a - a}{4} = \frac{a}{2} \]

\[ \frac{x}{x - 1} + \frac{1}{x + 2} = \frac{(x - 1)(x + 2) + (x - 1)(x + 2)}{(x - 1)(x + 2)} = \frac{2x - 1}{x + 1} \]

19 to 26: Find the sum or difference as indicated (reduce if possible):

19. \( \frac{1}{y} + \frac{2}{y} = \frac{3}{y} \)

20. \( \frac{3}{x - 3} - \frac{x}{x - 3} = \frac{3 - x}{x - 3} \)

21. \( \frac{b - a}{b + a} - \frac{a - b}{b + a} = 0 \)

22. \( \frac{x}{x - 1} + \frac{x}{1 - x} = \frac{x}{x - 1} \)

23. \( \frac{1}{a} - \frac{1}{2a} = \frac{1}{2a} \)

24. \( \frac{3x - 2}{x - 2} - \frac{2}{x - 2} = \frac{3x - 4}{x - 2} \)

25. \( \frac{2x - 1}{x + 1} - \frac{2x - 1}{x - 2} = \frac{4x - 5}{x - 2} \)

26. \( \frac{(x - 1)(x - 2)}{(x - 3)(x - 1)} = \frac{(x - 1)(x - 2) - (x - 1)(x - 3)}{(x - 3)(x - 1)} = \frac{4}{x - 3} \)

E. Multiplying fractions: multiply the tops, multiply the bottoms, reduce if possible.

Example:
\[ \frac{3}{1} \cdot \frac{2}{5} = \frac{6}{10} = \frac{3}{5} \]

Example:
\[ \frac{3(x + 1)}{x - 2} \cdot \frac{x^2 - 4}{x - 2} = \frac{3(x + 1)(x + 2)(x - 1)(x - 1)}{(x - 2)^2} = \frac{3x + 6}{x - 2} \]

27. \( \frac{2}{7a} \cdot \frac{ab}{12} = \frac{a}{42} \)

28. \( \frac{3(x + 4)}{x^2 - 1} \cdot \frac{5y^3}{x^2 - 16} = \frac{15y^3(x + 4)}{x^2 - 16} \)

29. \( \frac{(a + b)^3}{x - y} \cdot \frac{(p - 5)^2}{(5 - p)(a + b)^2} = \frac{(a + b)^3(p - 5)^2}{(x - y)(5 - p)(a + b)^2} \)

F. Dividing fractions: a nice way to do this is to make a compound fraction and then multiply the top and bottom (of the big fraction) by the LCM of both:

\[ \frac{a}{b} + \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{d} + \frac{c}{c} \cdot \frac{d}{d} = \frac{ad}{bd} \]

Example:
\[ \frac{7}{3} \div \frac{1}{2} = \frac{7}{3} \cdot \frac{2}{1} = \frac{14}{3} = \frac{42}{1} = 42 \]

Example:
\[ \frac{5x}{2y} \div \frac{2x}{2y} = \frac{5x}{2y} \cdot \frac{2y}{2x} = \frac{5x \cdot 2y}{4y} = \frac{5x}{4y} \]

30. \( \frac{3}{a} \div \frac{b}{3} = \frac{3x + 7}{x - 3} \)

31. \( \frac{2}{a} \div \frac{9}{a} = \frac{2a}{a} = 2 \)

32. \( \frac{a}{b} \div \frac{b}{a} = \frac{a^2}{b^2} \)

33. \( \frac{2a}{b} \div \frac{a}{b} = \frac{2a^2}{b^2} \)

Answers:
1. \( \frac{1}{4} \)
2. \( \frac{2}{5} \)
3. \( \frac{3}{4} \)
4. \( \frac{2a}{5b} \)
5. \( \frac{2x + 3}{3} \)
6. \( \frac{5x + b}{5a + d} \)
7. \( \frac{1}{3} \)
8. \( \frac{4x + 21}{x + 2} \)
9. \( \frac{a}{2} \)
10. \( \frac{3}{7} \)
11. \( \frac{1}{x - 1} \)
12. \( \frac{1 + b}{2 - a} \)
13. \( \frac{1}{2} \)
14. \( \frac{3}{8} \)
15. \( \frac{1}{x} \)
16. \( \frac{1}{x + 1} \)
17. \( \frac{1}{x + 2} \)
18. \( \frac{5x + 1}{x + 1} \)
19. \( \frac{3x^2 + 2x}{x + 1} \)
20. \( \frac{6}{7} \)
21. \( \frac{2a + 2b}{x + 1} \)
22. \( 0 \)
23. \( 1 \)
24. \( \frac{2x + x}{x + 1} \)
25. \( \frac{9x + 1}{x + 1} \)
26. \( 0 \)
27. \( \frac{a}{b} \)
28. \( \frac{x}{y} \)
29. \( \frac{a + b}{x - y} \)
30. \( \frac{2}{x - 7} \)
31. \( \frac{1}{x - y} \)
32. \( \frac{x^2 - y}{y - 2a} \)
33. \( \frac{3a - 2b}{y} \)
Precalculus Diagnostic Test Practice

Topic 2: Operations with exponents and radicals

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don’t get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Definitions of powers and roots:

1-20: Find the value:

1. $2^3 =$
2. $3^2 =$
3. $-4^2 =$
4. $(−4)^2 =$
5. $0^b =$
6. $1^b =$
7. $\sqrt{64} =$
8. $\sqrt[3]{64} =$
9. $\sqrt[3]{64} =$
10. $−\sqrt{49} =$
11. $\sqrt[5]{125} =$
12. $\sqrt[5]{2^5} =$
13. $\sqrt[5]{(-5)^2} =$
14. $\sqrt[5]{x^5} =$
15. $\sqrt[5]{a^3} =$
16. $\sqrt[5]{b^3} =$
17. $\sqrt[6]{a^6} =$
18. $(\sqrt[3]{2})^4 =$
19. $\sqrt[3]{64a^3} =$
20. $\sqrt{x} \cdot \sqrt{x} =$

21 to 30: Find $x$:

21. $2^3 \cdot 2^4 = 2^x$
22. $\frac{3^3}{2^4} = 2^x$
23. $\frac{3^4}{3} = \frac{1}{3^x}$
24. $\frac{5^2}{5^5} = 5^x$
25. $(3^4)^{\frac{1}{2}} = 2^x$
26. $\frac{8}{x} = 2^x$
27. $a^3 \cdot a = a^{x+3}$
28. $\frac{b^{10}}{b^5} = b^x$
29. $\frac{c^4}{c^3} = c^x$
30. $\frac{a^{2y} - 2}{a^{2y} - 3} = a^x$

31 to 43: Find the value:

31. $7x^0 =$
32. $3x^4 =$
33. $23 \cdot z^3 =$
34. $5^5 =$
35. $5^0 =$
36. $(-3)^3 \cdot 3^2 =$
37. $x^{c+3} \cdot x^{c-3} =$
38. $2^x \cdot 4^x - 1 =$
39. $\frac{a^x + 3}{a^{x-3}} =$
40. $\frac{a^{-x}}{a^{x-1}} =$
41. $\frac{e^{-x} - 3}{6x^{-1}} =$
42. $(\frac{a^x}{3})^{x-3} =$
43. $\frac{x^{3x} - 2}{x^{3x} - 3} =$

44 to 47: Write given two ways:

44. $\frac{d^4}{d^4}$
45. $(\frac{3x^3}{y})^2$
46. $(\frac{a^3}{b^5})^3$
47. $\frac{x^2 \cdot 3 - 1}{y^2 \cdot 5 - 3}$

C. Laws of rational exponents, radicals. Assume all radicals are real numbers:

I. If $r$ is a positive integer, $p$ is an integer, and $a > 0$, then

$s^p/r = \sqrt[p]{s^p} = (\sqrt[p]{s})^p$

II. $(ab)^{1/r} = a^{1/r} \cdot b^{1/r}$

III. $(a/b)^{1/r} = a^{1/r} / b^{1/r}$

IV. $r^{1/r} = a^{1/r} = \sqrt[r]{a}$

48 to 53: Write as a radical:

48. $\sqrt[3]{2}$
49. $\sqrt[4]{3}$
50. $(\frac{1}{2})^{1/3}$
51. $\sqrt[3]{2}$
52. $\sqrt[2]{3}$
53. $(2x)^{1/2}$

54 to 57: Write as a fractional power:

54. $\sqrt[3]{3}$
55. $\sqrt[2]{3}$
56. $\sqrt[3]{a}$
57. $\frac{1}{\sqrt[3]{a}}$

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One of a series of worksheets designed to provide remedial practice. Coordinated with topics on diagnostic tests supplied by the Mathematics Diagnostic Testing Project, Gayley Center Suite 304, UCLA, 405 Hilgard Ave., Los Angeles, CA 90024.
8. Rationalization of denominators:

\[
\text{example: } \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{6}}{3} \\
\text{example: } \frac{1}{\sqrt{2}} = \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2} \\
\text{example: } \sqrt{3} - 1 = \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{2}{\sqrt{3} + 1} \\
\]

79 to 87: Simplify:

79. \( \sqrt{31} = \)

80. \( \sqrt[3]{3} = \)

81. \( \sqrt[3]{1} = \)

82. \( \sqrt[8]{8} = \)

83. \( \sqrt[2]{2} = \)

84. \( \sqrt[2]{2} + \sqrt[2]{2} = \)

85. \( \sqrt[2]{2} + 1 = \)

86. \( \sqrt[2]{3} - 2 = \)

87. \( \sqrt[3]{3} + 2 = \)

Answers:
1. \( 8 \)
2. \( 9 \)
3. \(-16 \)
4. \( 16 \)
5. \( 0 \)
6. \( 1 \)
7. \( 8 \)
8. \( 4 \)
9. \( 2 \)
10. \(-7 \)
11. \(-5 \)
12. \( 5 \)
13. \( 6 \)
14. \( x \text{ if } x \geq 0 \)
15. \( a \)
16. \( 1/2 \)
17. \(-2 \)
18. \(-16/\sqrt{2} \)
19. \( 3 \sqrt{3} \)
20. \( 7 \)
21. \( 7 \)
22. \(-1 \)
Precalculus Diagnostic Test Practice
Topic 3: Linear equations and inequalities

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don’t get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Solving linear equations: add or subtract the same thing on each side of the equation, or multiply or divide each side by the same thing, with the goal of getting the variable alone on one side. If there are one or more fractions, it may be desirable to eliminate them by multiplying both sides by the common denominator. If the equation is a proportion, you may wish to cross-multiply.

1 to 15: Solve:

1. \( 2x = 9 \)
2. \( 3 = \frac{6x}{5} \)
3. \( 3x + 7 = 6 \)
4. \( \frac{x}{3} = 5 \)
5. \( 5 - x = 9 \)
6. \( x + \frac{2x}{3} = 9 + 1 \)
7. \( 3x - x = 6 \)
8. \( \frac{x - 1}{1} = \frac{6}{7} \)
9. \( x - 4 = \frac{x}{2} + 1 \)
10. \( \frac{3x}{2x + 1} = \frac{5}{2} \)
11. \( 6 - 4x = x \)
12. \( \frac{3x}{2x + 1} = \frac{4}{3} \)
13. \( \frac{x + 3}{2x - 1} = 2 \)
14. \( 7x - 5 = 2x + 10 \)
15. \( \frac{1}{3} = \frac{x}{x + 8} \)

B. Solving a pair of linear equations: the solution consists of an ordered pair, an infinite number of ordered pairs, or no solution.

16 to 23: Solve for the common solution(s) by substitution or linear combinations:

16. \( x + 2y = 7 \)
17. \( x + y = 5 \)
18. \( 2x - y = -9 \)
19. \( 2x - y = 1 \)
20. \( 2x - 3y = 5 \)
21. \( 4x - 1 = y \)
22. \( x + y = 3 \)
23. \( 2x - y = 3 \)
24. \( x = 3 + 1 \)
25. \( y = \frac{1}{2}x - 3 \)
26. \( 2y = 4x + 8 \)
27. \( x - y = -1 \)
28. \( x = -3y + 2 \)

To find an equation of a non-vertical line, it is necessary to know its slope and one of its points. Write the slope of the line thru \((x, y)\) and the known point, then write an equation which says that this slope equals the known slope.

example: Find an equation of the line thru \((-4, 1)\) and \((-2, 0)\).
Slope = \(\frac{1 - 0}{-4 - 2} = \frac{1}{-2} = \frac{-1}{2} \)
Using \((-2, 0)\) and \((x, y)\), slope = \(\frac{y - 0}{x - 2} = \frac{1}{-2} \); cross-multiply, get \(-2y = x + 2\), or \(y = \frac{-1}{2}x + 1\)

29 to 33: Find an equation of line:
29. thru \((-3, 1)\) and \((-1, -4)\)
30. thru \((0, -2)\) and \((-3, -5)\)
31. thru \((3, -4)\) and \((5, -1)\)
32. thru \((5, 0)\), with slope \(-1\)
33. thru \((0, -5)\), with slope \(2/3\)

A vertical line has no slope, and its equation can be written so it looks like \(x = k\) (where \(k\) is a number). A horizontal line has zero slope, and its equation looks like \(y = k\).

example: Graph on the same graph:
\(x + 3 = 1\) and \(1 + y = 3\).
The first equation is \(x = -1\).
The second is \(y = -4\).

34 to 35: Graph line on the same axes for:
34. the line thru \((-1, 4)\) and \((-1, 2)\)
35. horizontal line thru \(4, -1\)
D. Analytic geometry of two linear equations: two distinct lines in a plane are either parallel or intersecting. They are parallel if and only if they have the same slope, and hence the equations of the lines have no common solutions. If the lines have unequal slopes, they intersect in one point and their equations have exactly one common solution. (They are perpendicular if their slopes are negative reciprocals, or one is horizontal and the other is vertical.) If one equation is a multiple of the other, each equation has the same graph and every solution of one equation is a solution of the other.

36 to 43: For each pair of equations in problems 16 to 23, tell whether the lines are parallel, perpendicular, intersecting but not perpendicular, or the same line:

36. Problem 16 40. Problem 20
37. 17 41. 21
38. 18 42. 22
39. 19 43. 23

E. Linear inequalities:

example: One variable graph: solve and graph on number line: \(1 - 2x \leq 7\) (This is an abbreviation for \([x: 1 - 2x \leq 7]\))

Subtract 1, get \(-2x \leq 6\)
Divide by \(-2\), \(x \geq -3\)

Graph: \([-4 \rightarrow -3 \rightarrow -2 \rightarrow -1 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 3\)

44 to 50: Solve and graph on no. line:

44. \(x - 3 > 4\) 48. \(4 - 2x < 6\)
45. \(l < 2\) 49. \(5 - x > x - 3\)
46. \(2x + 1 \leq 6\) 50. \(x > 1 + 4\)
47. \(3 < x - 3\)

example: Two variable graph: graph solution on number plane: \(x - y \geq 3\) (This is an abbreviation for \([x: y: x - y \geq 3]\). Subtract \(x\), multiply by \(-1\), get \(y \leq x - 3\).

Graph \(y = x - 3\), but draw a dotted line, and shade the side where \(y < x - 3\).

1 to 56: Graph on number plane:

1. \(y < 3\) 54. \(x < y + 1\)
2. \(y > x\) 55. \(x + y < 3\)
3. \(y \geq \frac{1}{2} x + 2\) 56. \(2x - y > 1\)

F. Absolute value equations and inequalities:

example: \(|3 - x| = 2\)
Since the absolute value of both 2 and -2 is 2, \(3 - x\) can be either 2 or -2. Write these two equations and solve each:

\(3 - x = 2\) or \(3 - x = -2\)
\(-x = -1\) or \(-x = -5\)
\(x = 1\) or \(x = 5\)

Graph: \([-1 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6\)

57 to 61: Solve and graph on no. line:

57. \(|x| = 3\) 60. \(|2 - 3x| = 0\)
58. \(|x| = -1\) 61. \(|x + 2| = 1\)
59. \(|x - 1| = 3\)

example: \(|3 - x| < 2\)
The absolute value of any number between -2 and 2 (exclusive) is less than 2. Write this inequality and solve:

\(-2 < 3 - x < 2\). Subtract 3, multiply by -1, get \(5 > x > 1\). (Note that this says \(x > 1\) and \(x < 5\))

Graph: \([-4 \rightarrow -3 \rightarrow -2 \rightarrow -1 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 3\)

62 to 66: Solve and graph on no. line:

62. \(|x| < 3\) 65. \(1 \leq |x + 3|\)
63. \(3 < |x|\) 66. \(|5 - x| < 5\)
64. \(|x + 3| < 1\)

23. \((a, 2a - 3)\), \(a\) is any number; infinite no. of solutions
24. \(a = -1\)
25. \(b = 2\)
26. \(z = 2 + 3\)
27. \(z = 1\)
28. \(b = 2\)
29. \(y = 5/2 x - 3\)
30. \(y = x + 2\)
31. \(x = 1\)
32. \(y = x + 8\)
33. \(y = 2x - 5\)
34. \(x = 1\)
35. \(y = 1\)
36. \(x = 3\)
37. \(x = 4\)
38. \(x = 5\)
39. \(x = 6\)
40. \(x = 7\)
41. \(x = 8\)
42. \(x = 9\)
43. \(x = 10\)

44. \(x + 7\)
45. \(x + 1/2\)
46. \(x + 5/2\)
47. \(x + 1\)
48. \(x + 4\)
49. \(x + 5\)
50. \(x + 6\)
51. \(x + 7\)
52. \(x + 8\)
53. \(x + 9\)
54. \(x + 10\)
55. \(x + 11\)
56. \(x + 12\)
57. \(x + 13\)
58. \(x + 14\)
59. \(x + 15\)
60. \(x + 16\)
61. \(x + 17\)
62. \(x + 18\)
63. \(x + 19\)
64. \(x + 20\)
65. \(x + 21\)
66. \(x + 22\)

44. \(b = 2\)
45. \(b = 3\)
46. \(b = 4\)
47. \(b = 5\)
48. \(b = 6\)
49. \(b = 7\)
50. \(b = 8\)
51. \(b = 9\)
52. \(b = 10\)
53. \(b = 11\)
54. \(b = 12\)
55. \(b = 13\)
56. \(b = 14\)
57. \(b = 15\)
58. \(b = 16\)
59. \(b = 17\)
60. \(b = 18\)
61. \(b = 19\)
62. \(b = 20\)
63. \(b = 21\)
64. \(b = 22\)
65. \(b = 23\)
66. \(b = 24\)

44. \(a = 1\)
45. \(a = 2\)
46. \(a = 3\)
47. \(a = 4\)
48. \(a = 5\)
49. \(a = 6\)
50. \(a = 7\)
51. \(a = 8\)
52. \(a = 9\)
53. \(a = 10\)
54. \(a = 11\)
55. \(a = 12\)
56. \(a = 13\)
57. \(a = 14\)
58. \(a = 15\)
59. \(a = 16\)
60. \(a = 17\)
61. \(a = 18\)
62. \(a = 19\)
63. \(a = 20\)
64. \(a = 21\)
65. \(a = 22\)
66. \(a = 23\)

Answer:
1. 9/2
2. 5/2
3. -1/3
4. 1/5
5. -4
6. 5/3
7. 2
8. 13
9. 10
10. -5/4
11. 6/5
12. -6/5
13. 5/3
14. 3
15. 4
16. (9, -11)
17. (1, 4)
18. (4, 23)
19. (-4, -9)
20. (26/19, -13/19)
21. (1/4, 0)
22. no solution
Precalculus Diagnostic Test Practice
Topic 4: Polynomials and polynomial equations

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don’t get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Solving quadratic equations by factoring; if \( ab = 0 \), then \( a = 0 \) or \( b = 0 \).

**Example:** \((x + 2)(x - 3) = 0\) then \((x - 3) = 0 \) or \((x + 2) = 0\) and thus \(x = 3\) or \(x = -2\)

Note: there must be a zero on one side of the equation to solve by the factoring method.

**Example:** \(6x^2 = 3x\)
Rewrite: \(6x^2 - 3x = 0\)
Factor: \(3x(2x - 1) = 0\)
So \(3x = 0\) or \((2x - 1) = 0\)
Thus \(x = 0\) or \(x = 1/2\)

1 to 12: Solve by factoring:
1. \(x(x - 3) = 0\)
2. \(x^2 - 2x = 0\)
3. \(2x^2 = x\)
4. \(3x + 4 = 0\)
5. \((x + 2)(x - 3) = 0\)
6. \((2x + 1)(3x - 2) = 0\)
7. \(x^2 = x - 6 = 0\)
8. \(x^2 = 2 - x\)
9. \(6x^2 = x + 2\)
10. \(x^2 + x = 6\)
11. \(9 + x^2 = 6x\)
12. \(1 - x = 2x^2\)

B. Monomial factors: the distributive property says \(ab + ac = a(b + c)\)

**Example:** \(x^2 - x = x(x - 1)\)
**Example:** \(4x^2y + 6xy = 2xy(2x + 3)\)

13 to 17: Factor:
13. \(a^2 + ab\)
14. \(a^3 - a^2b + ab^2\)
15. \(-4xy + 10x^2\)
16. \(x^2 - y^2 = x\)
17. \(6x^3y^2 - 9x^3y = 6x^2y(2x - 3)\)

C. Factoring \((x - a)(x + b)^2 + (x - a)(x + c): the distributive property says \(jm + jn = j(m + n)\). Compare this equation with the following:
\((x + 1)(x + 3)^2 + (x + 1)(x - 4) = (x + 1)((x + 3)^2 + (x - 4))\)
Note that \(j = x + 1\), \(m = (x + 3)^2\), and \(n = (x - 4)\), and we get \((x + 1)(2^2 + 6x + 9 + x - 4) = (x + 1)(2^2 + 7x + 5)\)

18 to 20: Find \(P\) which completes the equation:
18. \((x - 2)(x - 1)^2 - (x - 2)(x + 3) = (x - 2)(\ P \)
19. \((x + 1)(x - 3)^2 + (x - 4)(x - 3) = (x - 3)(\ P \)
20. \((2x - 1)(x + 1) - (2x - 1)^2(x + 3) = (2x - 1)(\ P \)

D. The quadratic formula: if a quadratic equation looks like \(ax^2 + bx + c = 0\), then the roots (solutions) can be found by using the quadratic formula:
\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

**Example:** \(3x^2 + 2x - 1 = 0\)
\[a = 3, b = 2, \text{ and } c = -1\]
\[x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-1)}}{2 \cdot 3} = \frac{-2 \pm \sqrt{4 + 12}}{6} = \frac{-2 \pm \sqrt{16}}{6}\]
\[x = \frac{-2 \pm 4}{6} = \frac{-2 \pm 4}{6} = -1 \text{ or } 2\]

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E. Quadratic inequalities:

\[ \text{example: Solve } x^2 - x < 6. \text{ First make one side zero: } x^2 - x - 6 < 0 \]

Factor: \((x - 3)(x + 2) < 0\).

If \((x - 3) = 0\) or \((x + 2) = 0\), then \(x = 3\) or \(x = -2\).

These two numbers \((3\text{ and }-2)\) split the real numbers into three sets (visualize the number line):

<table>
<thead>
<tr>
<th>(x)</th>
<th>((x - 3))</th>
<th>((x + 2))</th>
<th>((x - 3)(x + 2))</th>
<th>solution?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x &lt; -2)</td>
<td>negative</td>
<td>negative</td>
<td>no</td>
<td></td>
</tr>
<tr>
<td>(-2 &lt; x &lt; 3)</td>
<td>negative</td>
<td>positive</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>(x &gt; 3)</td>
<td>positive</td>
<td>positive</td>
<td>no</td>
<td></td>
</tr>
</tbody>
</table>

Therefore, if \((x - 3)(x + 2) < 0\), then \(-2 < x < 3\).

Note that this solution means that \(x > -2\) and \(x < 3\).

\[ \text{25 to 29: Solve, and graph on number line:} \]

25. \(x^2 - x - 6 > 0\)
26. \(x^2 + 2x < 0\)
27. \(x^2 - 2x < -1\)
28. \(x > x^2\)
29. \(2x^2 + x - 1 > 0\)

F. Completing the square:

\(x^2 + bx\) will be the square of a binomial when \(c\) is added, if \(c\) is found as follows; find half the coefficient of \(x\), and square it—this is \(c\). Thus

\[ c = \left(\frac{b}{2}\right)^2 = \frac{b^2}{4}, \text{ and } x^2 + bx + c = x^2 + bx + \frac{b^2}{4} = (x + \frac{b}{2})^2 \]

\[ \text{example: } x^2 + 5x \]

Half of 5 is \(\frac{5}{2}\), and \((\frac{5}{2})^2 = \frac{25}{4}\), which must be added to complete the square: \(x^2 + 5x + \frac{25}{4} = (x + \frac{5}{2})^2\)

If the coefficient of \(x^2\) is not 1, factor so it is.

\[ \text{example: } 3x^2 - x = 3(x^2 - \frac{1}{3}x) \]

Half of \(-\frac{1}{3}\) is \(-\frac{1}{6}\), and \((-\frac{1}{6})^2 = \frac{1}{36}\), so

\[ (x^2 - \frac{1}{3}x + \frac{1}{36}) = (x - \frac{1}{6})^2, \text{ and} \]

\[ 3(x^2 - \frac{1}{3}x + \frac{1}{36}) = 3x^2 - x + \frac{3}{36} \]

Thus \(\frac{x}{36}\) (or \(\frac{1}{12}\)) must be added to \(3x^2 - x\) to complete the square.

30 to 33: Complete the square, and tell what must be added:

30. \(x^2 - 10x\)
32. \(x^2 - \frac{3}{2}x\)
31. \(x^2 + x\)
33. \(2x^2 + 8x\)

G. Graphing quadratic functions:

34 to 40: Sketch the graph:

34. \(y = x^2\)
35. \(y = -x^2\)
36. \(y = x^2 + 1\)
37. \(y = x^2 - 3\)
38. \(y = (x + 1)^2\)
39. \(y = (x - 2)^2 - 1\)
40. \(y = (x + 2)(x - 1)\)

Answers:

1. 0, 3
2. 0, 2
3. 0, 1/2
4. 0, -4
5. -2, 3
6. -1/2, 2/3
7. 3, -2
8. -2, 1
9. 2/3, -2/3
10. -3, 2
11. 3
12. -1, 1/2
13. 4x + 1
14. \((x^2 - 4x + 4)^2\)
15. \(2x^2 - 2x + 3x\)
16. \(y(x - y)\)
17. \(3x^2 - 3y^2\)
18. \(x^2 + x - 2\)
19. \(x^2 - 2x - 16\)
20. \(2x^2 - 4x + 4\)
21. \(-2, 3\)
22. \(-1, 3\)
23. \(-6, 4\)
24. \(-6, 4\)
25. \(-2, 2\) or \(x = 1 \frac{1}{2}\)
26. \(-2 < x < 0\)
27. no solution; no graph
28. \(0 < x < 1\)
29. \(x = -1\) or \(x = 1\)
30. \((x - 5)^2\), add 25
31. \((x - 1)^2\), add 1/4
32. \((x - 3/2)^2\), add 9/16
33. \((2x + 2)^2\), add 8
34. \(x^2 + 5x + 25\)
35. \(x^2 + 5x - 6\)
36. \(x^2 + 3x - 2\)
37. \(x^2 - 1\)
38. \(x^2 - 2\)
39. \(x^2 + 2\)
40. \(x^2 - 4\)
Topic 5: Functions

A. What functions are and how to write them: the area of a square depends on the side length \( s \), and we mean that given \( s \), we can find the area \( A \) for that value of \( s \). The side and area can be thought of as an ordered pair: \((s, A)\). For example, \((5, 25)\) is such an ordered pair. A function can be thought of in many ways; one very useful way is to think of a function as a set of ordered pairs with one restriction; no two different ordered pairs may have the same first element. Thus \[ (s, A); A \text{ is the area of the square with side length } s \] is a function consisting of an infinite set of ordered pairs.

Another way to look at a function is as a rule; for example, \( A = s^2 \) is the rule for finding the area of a square, given a side. The area depends on the given side and we say the area is a function of the side.

\( A(f(s)) \) is read 'A is a function of s', or 'A = f of s'. There are many functions of \( s \). The one here is \( s^2 \). We write this \( f(s) = s^2 \) and can translate: 'the function of \( s \) we're talking about is \( s^2 \). Sometimes we write \( A(s) = s^2 \). This says the area is a function of \( s \), and specifically, it is \( s^2 \).

In some relations, as \( x^2 + y^2 = 25 \), \( y \) is not a function of \( x \), since both \((3, 4)\) and \((3, -4)\) make the relation true.

1 to 7: Tell whether or not each set of ordered pairs is a function:

1. \([(1, 3), (-1, 3), (0, -1)]\)
2. \([(3, 1), (3, -1), (-1, 0)]\)
3. \([(0, 5)]\)
4. \([(x, y); y = x^2 \text{ and } x \text{ is any real number}]\)
5. \([(x, y); x = y^2 \text{ and } y \text{ is any real number}]\)
6. \([(x, y); y = x - 3 \text{ and } x \text{ is any real number}]\)
7. \([(x, y); y = 4 \text{ and } y \text{ is any real number}]\)
8 to 11: Is \( y \) a function of \( x \)?

8. \( y = \sqrt{x} + x^2 \)
9. \( y^2 = x^2 \)
10. \( x + y^2 = 36 \)
11. \( y = \sqrt{x} \)

B. Function values and substitution: if

\( A(s) = s^2 \), \( A(3) \), read 'A of 3', means replace every \( s \) in \( A(s) = s^2 \) with 3, and find the area when \( s \) is 3. When we do this, we find \( A(3) = 3^2 = 9 \).

Examples: \( g(x) \) is given: \( y = g(x) = mx^2 \)

\( g(3) = m \cdot 3^2 = 9m \)
\( g(7) = m \cdot 7^2 = 49m \)
\( g(s) = ms^2 \)
\( g(x + h) = m(x + h)^2 = mx^2 + 2m x h + mh^2 \)

12. Given \( y = f(x) = 3x - 2 \); complete these ordered pairs:

\((3, 1), (0, -1), (1/2, -5/2), (-1, 10), (1, -1), (-1, -5)\)

13 to 17: Given \( f(x) = \frac{x}{x+1} \)

Find:

13. \( f(1) = \)
14. \( f(-2) = \)
15. \( f(0) = \)
16. \( f(-1) = \)
17. \( f(x - 1) = \)

g(x) = x - 3 \), \( f(g(x)) \) is read \( 'f' \) of \( g \) of \( x \)' and means replace every \( x \) in \( f(x) = x^2 \) with \( g(x) \) giving \( f(g(x)) = (g(x))^2 \), which equals \((x - 3)^2 = x^2 - 6x + 9 \)

Example: \( g(f(1/2)) = (g(1/2))^2 \)

\( = g(1/4) = \frac{1}{16} \) - 3 = -2 \frac{1}{16} \)

C. Composition of functions

18 to 26: Use \( f \) and \( g \) as above:

18. \( g(f(x)) = \)
19. \( f(g(1)) = \)
20. \( g(g(x)) = \)
21. \( f(x) + g(x) = \)
22. \( f(x) - g(x) = \)

Example: \( k(x) = x^2 - lx \), for what \( x \) is \( k(x) = 0 \)?

If \( k(x) = 0 \), then \( x^2 - lx = 0 \) and since \( x^2 - lx = (x - l)^2 \), \( x \) can be either 0 or \( l \). (These values of \( x \) 0 and \( l \) are called 'zeros of the function', because each makes the function zero.)

27 to 30: Find \( x \) so:

27. \( k(x) = -4 \)
28. \( k(x) = 5 \)
29. \( x \) is a zero of \( x(x + 1) \)
30. \( x \) is a zero of \( (2x^2 - x - 3) \)

D. Graphing functions: an easy way to tell whether a relation between two variables is a function or not is by graphing it. If a vertical line can be drawn which has two or more points in common with the graph, the relation is not a function. If no vertical line touches the graph more than once, then it is a function.

Example: \( x = y^2 \)

Not a function

Vertical line hits it more than once.

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31 to 39: Tell whether or not each of the following is a function:

Since $y = f(x)$, the values of $y$ are the values of the function which correspond to specific values of $x$. The heights of the graph above (or below) the $x$-axis are the values of $y$ and so also of the function. Thus for this graph, $f(3)$ is the height (value) of the function at $x = 3$ and the value is 2:

At $x = -3$, the value (height) of $f(x)$ is zero; in other words, $f(-3) = 0$. Note that $f(3) 
eq f(-3)$, since $3 > 0$, and that $f(0) < f(-1)$, since $f(-1) = 1$ and $f(0) < 1$.

40 to 44: For this graph, tell whether the statement is true or false:

40. $g(-1) = g(0)$
41. $g(0) = g(3)$
42. $g(1) > g(-1)$
43. $g(-2) > g(1)$
44. $g(2) < g(0) < g(4)$

45 to 54: Sketch the graph:

45. $f(x) = -x$
46. $y = 4 - x$
47. $y = |x|$
48. $y = |x - 2|$
49. $y = x^2 + 3x - 4$
50. $y = \frac{x}{x}$
51. $f(x) = \frac{-2}{x}$
52. $y = \frac{1}{x - 2}$
53. $y = \frac{1}{x}$
54. $y = \frac{3}{x - 1}$

Answers:
1. yes
2. no
3. yes
4. yes
5. no
6. yes
7. no
8. yes
9. no
10. no
11. yes
12. yes
13. $\frac{1}{2}$
14. 2
15. 0
16. none
17. $\frac{1}{x}$
18. $x^2 - 3$
19. 4
20. $x + 6$
21. $x^2 + x - 3$
22. $2x - 3x^2$
23. $x^2 + x - 3$
24. $\frac{x}{x}$
25. $x^2 - 3$
26. $x^2 - bx + y$
A. Trig functions in right triangles: the sine ratio for an acute angle of a right triangle is defined to be the length of the opposite leg to the length of the hypotenuse.

Thus the sine ratio for angle B, abbreviated \( \sin B \), is \( \frac{b}{c} \).

The reciprocal of the sine ratio is the cosecant (csc), so \( \csc B = \frac{c}{b} \).

The other four trig ratios (all functions) are:
- **Cosine** = \( \cos \theta = \frac{adjacent}{hypotenuse} \)
- **Secant** = \( \sec \theta = \frac{1}{\cos \theta} = \frac{hypotenuse}{adjacent} \)
- **Tangent** = \( \tan \theta = \frac{opposite}{adjacent} \)
- **Cotangent** = \( \cot \theta = \frac{1}{\tan \theta} = \frac{adjacent}{opposite} \)

1 to 8: For this right triangle, give the following ratios:

1. \( \tan \theta \) =
2. \( \sin \theta \) =
3. \( \cos \theta \) =
4. \( \sin \theta \cdot \cos \theta \) =
5. \( \cos^2 \theta \), which means \( (\cos \theta)^2 \) =
6. \( 1 - \sin^2 \theta \) =
7. \( \cos \theta \) =
8. \( \sin \theta \) =

B. Circular trig definitions: given a circle with radius \( r \), centered on \((0, 0)\). To any point on the circle, draw the radius, making an angle \( \theta \) with the positive x-axis (\( \theta \) may be any real number, positive measure is counter-clockwise). The coordinates \((x, y)\) of point \( P \) together with radius \( r \) are used to define the functions: \( \sin \theta = \frac{y}{r} \), \( \cos \theta = \frac{x}{r} \), \( \tan \theta = \frac{y}{x} \), and the reciprocal functions as before. (Note that for \( 0 < \theta < \pi/2 \), these definitions agree with the right triangle definitions. Also note that \(-1 < \sin \theta < 1\), \(-1 < \cos \theta < 1\), and \(\tan \theta \) can be any real number.)

9 to 12: For the point \((-3, 4)\) on the above circle, give:

9. \( x = \) \( y = \) \( r = \)
10. \( \tan \theta = \)
11. \( \cos \theta = \)
12. \( \cot \theta \cdot \sin \theta = \)

Note that for any given value of a trig function, (in its range), there are infinitely many values of \( \theta \).

13 to 14: Find two positive and two negative values of \( \theta \) for which:

13. \( \sin \theta = -1 \)
14. \( \tan \theta = \tan 45^\circ \)

15 to 16: Given \( \sin \theta = 3/5 \) and \( \pi/2 < \theta < \pi \), then:

15. \( \tan \theta = \)
16. \( \cos \theta = \)

C. Pythagorean relations (identities):

\[ a^2 + b^2 = c^2 \] (or \( x^2 + y^2 = r^2 \))

above, can be divided by \( c^2 \) (or \( r^2 \))

to give \( a^2/c^2 + b^2/c^2 = a^2/c^2 \) or \( \sin^2 \theta + \cos^2 \theta = 1 \), called an identity because it is true for all values of \( \theta \) for which it is defined.

17 to 18: Get a similar identity by dividing \( a^2 + b^2 = c^2 \) by:

17. \( b^2 \)
18. \( a^2 \)

D. Similar triangles:

If \( \triangle ABC \sim \triangle DEF \), and if \( \tan A = 3/4 \),
then \( \tan D = 3/4 \) also, since \( \text{EF}/\text{DF} = \text{BC}/\text{AC} = 3/4 \).

19. Find \( DC \), given \( DB = 5 \) and \( \sin B = \frac{3}{5} \).

E. Radians and degrees:

For angle \( \theta \), there is a point \( P \) on the circle, and an arc from \( A \) counter-clockwise to \( F \). The length of the arc is \( \frac{\theta \cdot \pi \cdot r}{180} \), and the ratio of \( 360^\circ \) to \( \frac{\pi \cdot r}{180} \), where \( \theta \) is the number of degrees (and the ratio has no units). This is the radian measure associated with point \( F \). So \( F \) can be located two ways: by giving the central angle \( \theta \) in degrees, or in a number of radian to be wrapped around the circle from point \( A \) (the radian measure).

Converting: \( \text{radians} = \frac{\theta}{180} \cdot \text{degrees} \) or \( \text{degrees} = \frac{\theta}{\pi} \cdot \text{radians} \).

**Example:** \( \frac{\pi}{3} \) (radians) = \( \frac{\pi}{3} \cdot \frac{180}{\pi} = 60^\circ \)

**Example:** \( 420^\circ = 420 \cdot \frac{\pi}{180} = \frac{7\pi}{3} \) (radians)

(which means that it would take a little over 7 radian to wrap around the circle from \( A \) to \( 420^\circ \)).
20 to 23: Find the radian measure for a central angle of:
20. $360^\circ$ = $22\times 180^\circ$ = 22. $180^\circ$ = 21. $-45^\circ$ = 23. $217^\circ$ = 24 to 26: Find the degree measure which corresponds to radian measure of:
24. $\frac{3\pi}{2}$ = 26. $-\frac{7\pi}{6}$ = 25. $-3$ =

27 to 29: Find the following values by sketching the circle, central angle, and a vertical segment from point P to the x-axis. (Radian measure if no units are given.) Use no tables or calculator.
27. $\cos \frac{5\pi}{6}$ = 29. $\sin (-225^\circ)$ = 28. $\tan (-315^\circ)$ = 30 to 33: Sketch to evaluate without table or calculator:
30. $\sec 105^\circ$ = 32. $\sin \pi$ = 31. $\cot (-\frac{2\pi}{3})$ = 33. $\cos \frac{3\pi}{2}$ =

P. Trigonometric equations:

example: Solve, given that $0 \leq \theta < 2\pi$:

$\tan^2 \theta - \tan \theta = 0$. Factoring, we get
$\tan \theta (\tan \theta - 1) = 0$, which means that
$\tan \theta = 0$ or $\tan \theta = 1$. Thus $\theta = 0$ (degree or radians) plus any multiple of $180^\circ$ (or $\pi$), which is $n\cdot 180^\circ$ (or $n\cdot \pi$), or $\theta = 180^\circ + n\cdot 180^\circ$ (or $\frac{\pi}{2} + n\cdot \frac{\pi}{2}$).

Thus $\theta$ can be 0, $\pi$, $\pi$, or $2\pi$, which all check in the original equation.

34 to 39: Solve, for $0 \leq \theta < 2\pi$:

34. $\sin \theta = \cos \theta$
35. $\sin^2 \theta + \cos^2 \theta = 1$
36. $2 \sin \theta = 1 - \cos^2 \theta$
37. $\sin \theta \cot \theta = 1/2$
38. $\cos \theta \tan \theta = 1$
39. $\sin \theta \tan \theta = \sec \theta$

3. Graphs of trig functions: by finding values of $\sin x$ when $x$ is a multiple of $\pi/2$, we can get a quick sketch of $y = \sin x$. The sine is periodic (it repeats every $2\pi$, its period). $|\sin x|$ never exceeds 1, so the amplitude of $\sin x$ is 1, and we get this graph:

To graph $y = \sin 3x$, we note that for a given value of $x$, say $x = a$, the value of $y$ is found on the graph of $y = \sin x$ three times as far from the y-axis as $a$. Thus all points of the graph of $y = \sin 3x$ are found by moving $x$ each point of the $y = \sin x$ graph to 1/3 its previous distance from the y-axis, as shown. The new graph repeats 3 times in the period of $y = \sin x$, so the period of $y = \sin 3x$ is $2\pi/3$.

40 to 46: Sketch each graph and find its period and amplitude:
40. $y = \cos x$
41. $y = \cos 2x$
42. $y = \tan x$
43. $y = \tan \frac{x}{3}$
44. $y = -\frac{1}{4} \sin \frac{5}{3} x$
45. $y = \sin^2 x$
46. $y = 1 + \cos x$

H. Identities:

example: Find a formula for $\cos 2A$, given
$\cos (A + B) = \cos A \cos B - \sin A \sin B$.
Substitute $A$ for $B$; $\cos 2A = \cos (A + A) = \cos A \cos A - \sin A \sin A = \cos^2 A - \sin^2 A$

47 to 49: Use $\sin^2 x + \cos^2 x = 1$ and the above to show:
47. $\cos 2A = 2 \cos^2 A - 1$
48. $\cos 2A = 1 - 2 \sin^2 A$
49. $\cos^2 x = \frac{1}{2} (1 + \cos 2x)$

50. Given $\cos A = \frac{1}{2}$, and $\sin A = \sin A$, show that $\tan^2 x + 1 = \sec^2 x$

51. Given $\sin 2A = 2 \sin A \cos A$, show $\sin \frac{x}{2} \cos \frac{x}{2} = \frac{1}{2} \sin x$.
17. \( \log_3 (27 \cdot 3^{-4}) = x \)
18. \( \log (2x - 6) = \log (6 - x) \)
19. \( \log_{10} 64 = x \)
20. \( \sqrt[3]{5} = 5^x \)
21. \( \log_3 27 - \log_3 27 = x \)
22. \( \log_6 \sqrt[3]{30} = x \log_4 30 \)
23. \( 27^x = (\frac{1}{3})^3 \)
24. \( 4^{10} = 2^x \)
25. \( 3 \cdot 2^x = 4 \)
26 to 31: Find the value:
   26. \( 2^{10} = 1024 \)
   27. \( \log_4 4^{10} = 20 \)
   28. \( \log_6 6 = 1 \)
   29. \( 6^{-10} = \frac{1}{6^{10}} \)
   30. \( \log_{10} 7 = 0.8 \)
   31. \( \log_7 49 = 2 \)
32. Find \( \log_3 2 \), given:
   \( \log_{10} 3 = 0.477 \)
   \( \log_{10} 2 = 0.301 \)
33-34: Given \( \log_2 1024 = 10 \), find:
   33. \( \log_2 1024^5 = \)
   34. \( \log_2 \sqrt[5]{1024} = \)
35 to 40: Solve for \( x \) in terms of \( y \) and \( z \):
   35. \( y^3 = z^3 \)
   36. \( y^7 = y^3 \)
   37. \( x^3 = y \)
   38. \( 3^x = y \)
   39. \( 3 \cdot 2^x = 2^y \)
40. \( \log x^2 = 3 \log y \)
41. \( \log x = 2 \log y + \log z \)
42. \( 3 \log x = \log y \)
43. \( \log x = \log y + \log z \)
44. \( \log \sqrt{x} + \log \sqrt{y} = \log z + 2 \)
45. \( \log_2 3 = y \); \( \log_7 2 = x \)
46. \( y = \log_a 9 \); \( x = \log_a 3 \)

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B. Inverse functions and graphing: If 

\[ y = f(x) \text{ and } y = g(x) \] 

are inverse functions, then an ordered pair \((a, b)\) satisfies \(y = f(x)\) if only if the ordered pair \((b, a)\) satisfies \(y = g(x)\). In other words, \(f\) and \(g\) are inverses of each other if:

\[ f(a) = b \iff g(b) = a \]

To find the inverse of a function \(y = f(x)\):

1. Interchange \(x\) and \(y\).
2. Solve this equation for \(y\) in terms of \(x\), so \(y = g(x)\).
3. Then if \(g\) is a function, \(f\) and \(g\) are inverses of each other.

The effect on the graph of \(y = f(x)\) when \(x\) and \(y\) are switched is to reflect the graph over the \(45^\circ\)-line (bisecting quadrants I and III). This reflected graph represents \(y = g(x)\).

**Example:** Find the inverse of

\[ f(x) = \sqrt{x - 1}, \text{ or } y = \sqrt{x - 1} \]

1. Switch \(x\) and \(y\):

\[ x = \sqrt{y - 1} \quad \text{(note } y \geq 1 \text{ and } x \geq 0) \]

2. Solve for \(y\):

\[ x^2 = y - 1, \quad \text{so } y = x^2 + 1 \quad (x \geq 0 \text{ is still true}) \]

3. Thus \(g(x) = x^2 + 1\) (with \(x \geq 0\)) is the inverse function, and this graph:

Note that the \(f\) and \(g\) graphs are reflections of each other in the \(45^\circ\)-line, and that the ordered pair \((2, 5)\) satisfies \(g\) and \((5, 2)\) satisfies \(f\).

**Example:** Find the inverse of

\[ y = f(x) = 3^x \] 

and graph both functions on one graph:

1. Switch: \(x = y^2\)

2. Solve:

\[ \log_3 y = x \quad \text{is } g(x) \text{ of } f \]

To get the graph of \(g(x) = \log_3 x\), reflect the \(f\) graph over the \(45^\circ\)-line:

47 to 56: Find the inverse function and sketch the graphs of both:

47. \(f(x) = 3x - 2\)

48. \(f(x) = \log_2 (-x)\) (note that \(-x\) must be positive, which means \(x\) must be negative)

49 to 56: Sketch the graph:

49. \(y = x^4\)

50. \(y = 4^x\)

51. \(y = 4x - 1\)

52. \(y = \log_4 x\)

53. \(y = 4x - 3\)

54. \(y = -\log_4 x\)

55. \(y = 4x - 1\)

56. \(y = \log_4 (x - 1)\)
Topic E: Mathematical Modeling—word problems

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don’t get the answer given, check your work and look for mistakes.

1. $\frac{2}{3}$ of $\frac{1}{6}$ of $\frac{3}{4}$ of a number is 12. What is the number?

2. On the number line, points P and Q are 2 units apart. Q has coordinate $x$. What are the possible coordinates of P?

3. What is the number, which when multiplied by 32, gives 32 + 48?

4. If you square a certain number, you get 92. What is the number?

5. What is the power of 36 that gives 36$^{1/2}$?

6. Point X is on each of two given intersecting lines. How many such points X are there?

7. Point Y is on each of two given circles. How many such points Y are there?

8. Point Z is on each of a given circle and a given ellipse. How many such Z are there?

9. Point R is on the coordinate plane so its distance from a given point A is less than 4. Show in a sketch where R could be.

10. If the length of chord AB is $x$ and the length of GB is 16, what is AC?

11. If $AC = y$ and $GB = z$, how long is $AB$ (in terms of $y$ and $z$)?

12. This square is cut into two smaller squares and two non-square rectangles as shown. Before being cut, the large square had area $(a + b)^2$. The two smaller squares have areas $a^2$ and $b^2$. Find the total area of the two non-square rectangles. Do the areas of the i parts add up to the area of the original square?

13. Find $x$ and $y$:

14. In order to construct an equilateral triangle with an area which is 100 times the area of a given equilateral triangle, how long a side should be used?

15. $x$ and $y$ are numbers, and two $x$’s equal three $y$’s. Which of $x$ or $y$ must be larger?

16. What is the ratio of $x$ to $y$?

17. to 21: A plane has a certain speed in still air. In still air, it goes 1350 miles in 3 hours.

17. What is its (still air) speed?

18. How long does it take to fly 2000 mi.?

19. How far does the plane go in 6 hours?

20. If the plane flies against a 50 mph headwind, what is its ground speed?

21. If it has fuel for 7 1/2 hours of flying time, how far can it go against this headwind?

22 to 32: Georgie and Porgie bake pies. Georgie can complete 30 pies an hour.

22. How many can he make in one minute?

23. How many can he make in 10 minutes?

24. How many can he make in x minutes?

25. How long does he take to make 200 pies?

26 to 28: Porgie can finish 1/2 a pie an hour.

26. How many can she make in one minute?

27. How many can she make in 20 minutes?

28. How many can she make in x minutes?

29 to 32: If they work together, how many pies can they produce in:

29. 1 minute

30. x minutes

31. 30 minutes

32. 3 hours

33 to 41: A nurse needs to mix some alcohol solutions, given as a percent by weight of alcohol in water. Thus in a 3% solution, 3% of the weight would be alcohol. She mixes $x$ gm of 3% solution, $y$ gm of 10% solution, and 10 gm of pure water to get a total of 140 gm of a solution which is 8% alcohol.

33. In terms of $x$, how many gm of alcohol are in the 3% solution?

34. The $y$ gm of 10% solution would include how many gm of alcohol?

35. How many gm of solution are in the final mix (the 8% solution)?

36. Write an expression in terms of $x$ and $y$ for the total number of gm in the 8% solution contributed by the three ingredients (the 3%, 10%, and water).

37. Use your last two answers to write a “total grams equation.”

38. How many gm of alcohol are in the 8%?

39. Write an expression in terms of $x$ and $y$ for the total number of gm of alcohol in the final solution.

40. Use the last two answers to write a “total grams of alcohol equation.”

41. How many gm of each solution are needed?

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42. Half the square of a number is 18. What is the number?
43. If the square of twice a number is 61, what is the number?
44. Given a positive number \( x \). The square of a positive number \( y \) is at least \( 4 \) times \( x \). How small can \( y \) be?
45. Twice the square of half of a number is \( x \). What is the number?
46 to 48: Half of \( x \) is the same as one-third of \( y \).
46. Which of \( x \) and \( y \) is the larger?
47. Write the ratio \( x:y \) as the ratio of two integers.
48. How many \( x \)'s equal 30 \( y \)'s?
49 to 50: A gathering has twice as many women as men. If \( W \) is the number of women and \( M \) is the number of men, how much is correct: \( 2M = W \) or \( M = 2W \)?
50. Write the ratio \( \frac{W}{M} \) as the ratio of two integers.
51 to 53: If \( A \) is increased by 25\%, it equals \( B \).
51. Which is larger, \( B \) or the original \( A \)?
52. \( B \) is what percent of \( A \)?
53. \( A \) is what percent of \( B \)?
54 to 56: If \( C \) is decreased by 40\%, it equals \( D \).
54. Which is larger, \( D \) or the original \( C \)?
55. \( C \) is what percent of \( D \)?
56. \( D \) is what percent of \( C \)?
57 to 58: The length of a rectangle is increased by 25\% and its width is decreased by 40\%.
57. Its new area is what percent of its old area?
58. By what percent has the old area increased or decreased?
59 to 61: Your wage is increased by 20\%, then the new amount is cut by 20\% (of the new amount).
59. Will this result in a wage which is higher than, lower than, or the same as the original wage?
60. What percent of the original wage is this final wage?
61. If the above steps were reversed (20\% cut followed by 20\% increase), the final wage would be what percent of the original wage?
62 to 75: Write an equation for each of the following statements about real numbers and tell whether it is true or false:
62. The product of the squares of two numbers is the square of the product of the two numbers.
63. The square of the sum of two numbers is the sum of the squares of the two numbers.
64. The square of the square root of a number is the square root of the square of the number.